PHYSICS - 1998

PART - A

Directions

1. *Section I consists of 40 objective type questions.*
2. *This section should take about one hour to answer.*
3. *Each question in this section carries 2 marks.*

1. A transistor is used in common emitter mode as an amplifier, then :

(A) the base emitter junction is forward biased

(B) the base emitter junction is reverse biased

(C) the input signal is connected in series with the voltage applied to bias the base emitter junction

(D) the input signal is connected in series with the voltage applied to bias the base collector junction.

2. Water from a tap emerges vertically downwards with an initial speed of 1.0 m/s. The cross-sectional area of tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is steady, the cross-sectional area of stream 0.15 m below the tap is :

(A) 5.0×10^{-4} m^2 (B) 1.0×10^{-4} m^2

(C) 5.0×10^{-5} m^2 (D) 2.0×10^{-5} m^2

3. A real image of a distant object is formed by a planoconvex lens on its principal axis. Spherical aberration :

(A) is absent

(B) is smaller if the curved surface of the lens faces the object

(C) is smaller if the plane surface of the lens faces the object

(D) is the same whichever side of the lens faces the object.

4. Let $\bar{v}$, v_{rms} and v_p respectively denote the mean speed, root mean square speed and most probable speed of the molecules in an ideal monoatomic gas at absolute temperature T. The mass of a molecule is m. Then :

(A) no molecule can have a energy greater than $\sqrt{2}v_{rms}$

(B) no molecule can have speed less then $v_p/\sqrt{2}$

(C) $v_p < \bar{v} < v_{rms}$

(D) the average kinetic energy of a molecule is $\frac{3}{4} m{v_p}^2$

5. A vessel contains a mixture of one mole of oxygen and two moles of nitrogen at 300K. The ratio of the average rotational kinetic energy per O_2 molecule to per N_2 molecule is :

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) depends on the moment of inertia of the two molecules.

6. A string of length 0.4 m and mass 10^{-2} Kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time Δt. The minimum value of Δt, which allows constructive interference between successive pulses, is :

(A) 0.05 s (B) 0.10 s
(C) 0.20 s (D) 0.40 s

7. Two particles, each of mass m and charge q, are attached to the two ends of a light rigid rod of length 2R. The rod is rotated at constant angular speed about a perpendicular axis passing through its centre. The ratio of the magnitudes of the magnetic moment of the system and its angular momentum about the centre of the rod is :

(A) $q/2m$ (B) q/m
(C) $2q/m$ (D) $q/\pi m$

8. A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence 45°. The ray undergoes total internal reflection. If n is the refractive index of the medium with respect to air, select the possible value (s) of n from the following :

(A) 1.3 (B) 1.4
(C) 1.5 (D) 1.6

9. Let m_p be the mass of proton, m_n the mass of neutron. M_1 the mass of $^{20}_{10}Ne$ nucleus and M_2 the mass of $^{40}_{20}Ca$ nucleus. Then :

(A) $M_2 = 2M_1$ (B) $M_2 > 2M_1$
(C) $M_2 < 2M_1$ (D) $M_1 < 10(m_n + m_p)$

10. A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of the incident beam. At the first minimum of the diffraction pattern, the phase difference between the rays coming from the two edges of the slit is :

(A) 0 (B) $\pi/2$
(C) π (D) 2π

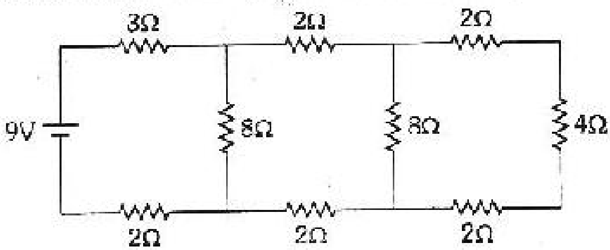
11. The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$ where n_1 and n_2 are the principal quantum numbers of two states. Assume the Bohr model to be valid. The time period of the electron in the intial state is eight times that in the final state. The possible values of n_1 and n_2 are :

(A) $n_1 = 4, n_2 = 2$ (B) $n_1 = 8, n_2 = 2$
(C) $n_1 = 8, n_2 = 1$ (D) $n_1 = 6, n_2 = 3$

12. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position, where the string is horizontal, is :

(A) $\sqrt{u^2 - 2gL}$ (B) $\sqrt{2gL}$
(C) $\sqrt{u^2 - gL}$ (D) $\sqrt{2(u^2 - gL)}$

13. In the circuit shown in the figure, the current through :



(A) the 3Ω resistor is 0.50 A
(B) the 3Ω resistor is 0.25 A
(C) the 4Ω resistor is 0.50 A
(D) the 4Ω resistor is 0.25 A

14. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at $x = 0$ and positive plate is at $x = 3d$. The slab is equidistant from the plates. The capacitor is given some charge. As x goes from 0 to 3d :

(A) the magnitude of the electric field remains the same
(B) the direction of the electric field remains the same
(C) the electric potential increases continuously
(D) the electric potential increases at first, then decreases and again increases.

15. The (x, y) coordinates of the corners of a square plate are (0, 0), (L, 0), (L, L) and (0, L). The edges of the plate are clamped and transverse standing waves are set up in it. If u (x, y) denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression (s) for u is (are) (a= positive constant) :

(A) $a \cos(\pi x/2L) \cos(\pi y/2L)$
(B) $a \sin(\pi x/L) \sin(\pi y/L)$
(C) $a \sin(\pi x/L) \sin(2\pi y/L)$
(D) $a \cos(2\pi x/L) \sin(\pi y/L)$

16. A force $\vec{F} = -K(y\hat{i} + x\hat{j})$ (where K is a positive constant) acts on a particle moving in the xy plane. Starting from the origin, the particle is taken along the positive x-axis to the point (a, 0) and then parallel to the y-axis to the point (a, a). The total work done by the force F on the particle is :

(A) $-2Ka^2$
(B) $2Ka^2$
(C) $-Ka^2$
(D) Ka^2

17. A small square loop of wire of side l is placed inside a large square loop of wire of side L ($L >> l$). The loops are coplanar and their centres coincide. The mutual inductance of the system is proportional to :

(A) l/L
(B) l^2/L
(C) L/l
(D) L^2/l

18. The half life of ^{131}I is 8 days. Given a sample of ^{131}I at time $t = 0$, we can assert that :

(A) no nucleus will decay before $t = 4$ days
(B) no nucleus will decay before $t = 8$ days

(C) all nuclei will decay before $t = 16$ days

(D) a given nucleus may decay at any time after $t = 0$

19. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of the gas in A is m_A and that in B is m_B. The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The changes in the pressure in A and B are found to be ΔP and $1.5\,\Delta P$ respectively. Then .

(A) $4\,m_A = 9m_B$ (B) $2m_A = 3m_B$

(C) $3m_A = 2m_B$ (D) $9m_A = 4m_B$

20. A given quantity of an ideal gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is :

(A) $\frac{2}{3}P$ (B) P

(C) $\frac{3}{2}P$ (D) 2P

21. A charge + q is fixed at each of the points $x = x_0$, $x = 3x_0$, $x = 5x_0$ ... ∞ on the x-axis and a charge −q is fixed at each of the points $x = 2x_0$, $x = 4x_0$, $x = 6x_0$ ... ∞. Here x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $Q/4\pi\epsilon_0 r$. Then the potential at the origin due to the above system of charges is :

(A) 0 (B) $\frac{q}{8\pi \epsilon_0 x_0 \ln 2}$

(C) ∞ (D) $\frac{q \ln (2)}{4\pi \epsilon_0 x_0}$

22. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to :

(A) I (B) $I \sin^2 \theta$

(C) $I \cos^2 \theta$ (D) $I \cos^2 (\theta/2)$

23. Two cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30 K, then the rise in temperature of the gas in B is :

(A) 30 K (B) 18 K

(C) 50 K (D) 42 K

24. A concave mirror is placed on a horizontal table with its axis directed vertically upwards. Let O be the pole of the mirror and C its centre of curvature. A point object is placed at C. It has a real image, also located at C. If the mirror is now filled with water, the image will be :

(A) real and will remain at C

(B) real and located at a point between C and ∞

(C) virtual and located at a point between C and 0
(D) real and located at a point between C and 0

25. A metal rod moves at a constant velocity in a direction perpendicular to its length. A constant uniform magnetic field exists in space in a direction perpendicular to the rod as well as its velocity. Select the correct statement (s) from the following :
(A) the entire rod is at the same electric potential
(B) there is an electric field in the rod
(C) the electric potential is highest at the centre of the rod and decreases towards its ends.
(D) the electric potential is lowest at the centre of the rod and increases towards its ends.

26. A positively charged thin metal ring of radius R is fixed in the xy plane with its centre at the origin O. A negatively charged particle P is released from rest at the point $(0, 0, z_0)$ where $z_0 > 0$. Then the motion of P is :
(A) periodic for all values of z_0 satisfying $0 < z_0 < \infty$
(B) simple harmonic for all values of z_0 satisfying $0 < z_0 \leq R$
(C) approximately simple harmonic provided $z_0 << R$
(D) such that P crosses 0 and continues to move along the negative z-axis towards $z = -\infty$

27. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth :
(A) The acceleration of S is always directed towards the centre of the earth
(B) The angular momentum of S about the centre of the earth changes in direction, but its magnitude remain constant.
(C) the total mechanical energy of S varies periodically with time
(D) The linear momentum of S remains constant in magnitude.

28. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A} \times \vec{L}$ where $\vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about that point. From this it follows that :
(A) $\frac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$ at all instants of time.
(B) the component of $\vec{L}$ in the direction of $\vec{A}$ does not change with time
(C) the magnitude of $\vec{L}$ does not change with time.
(D) $\vec{L}$ does not change with time.

29. During the melting of a slab of ice at 273K at atmospheric pressure :
(A) positive work is done by the ice-water system on the atmosphere.
(B) positive work is done on the ice-water system by the atmosphere.
(C) the internal energy of the ice water system increases.
(D) the internal energy of the ice-water system decreases.

30. In a p-n junction diode not connected to any circuit :

(A) the potential is the same everywhere

(B) the p-type side is at a higher potential than the n-type side

(C) there is an electric field at the junction directed from the n-side to the p-type side.

(D) there is an electric field at the junction directed from the p-type side to the n-type side

31. A spherical surface of radius of curvature R, separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at a point O and PO = OQ. The distance PC is equal to :

(A) 5R (B) 3R

(C) 2R (D) 1.5 R

32. A non-conducting solid sphere of radius R is uniformly charged. The magnitude of the electric field due to the sphere at a distance r from its centre :

(A) increases as r increases for $r < R$

(B) decreases as r increases for $0 < r < \infty$

(C) decreases as r increases for $R < r < \infty$

(D) is discontinuous at $r = R$

33. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is v/10, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10$ m/s, then λ and f are given by :

(A) $\lambda = 2\pi \times 10^{-2}$ m (B) $\lambda = 10^{-3}$ m

(C) $f = \frac{10^3}{2\pi}$ Hz (D) $f = 10^4$ Hz

34. A black body is at a temperature of 2880 K. The energy of radiation emitted by this object with wavelength between 499 nm and 500 nm is U_1, between 999 nm and 1000 nm is U_2 and between 1499 nm and 1500 nm is U_3. The Wein constant, $b = 2.88 \times 10^6$ nm-K. Then :

(A) $U_1 = 0$ (B) $U_3 = 0$

(C) $U_1 > U_2$ (D) $U_2 > U_1$

35. Let $[\epsilon_0]$ denote the dimensional formula of the permittivity of the vacuum and $[\mu_0]$ that of the permeability of the vacuum. If M = mass, L = length, T = time and I = electric current :

(A) $[\epsilon_0] = [M^{-1}L^{-3}T^2I]$ (B) $[\epsilon_0] = [M^{-1}L^{-3}T^4I^2]$

(C) $[\mu_0] = [MLT^{-2}I^{-2}]$ (D) $[\mu_0] = [ML^2T^{-1}I]$

36. The SI unit of the inductance, the henry can be written as :

(A) Weber/ampere (B) Volt-second/ampere

(C) Joule/(ampere)^2 (D) ohm-second

37. Two very long straight parallel wires carry steady currents I and I respectively. The distance between the wires is d. At a certain instant of time, a point charge q is at a point equidistant from the two wires in the plane of the wires. Its instantaneous velocity $\vec{v}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is :

(A) $\frac{\mu_0 Iqv}{2\pi d}$ (B) $\frac{\mu_0 Iqv}{\pi d}$

(C) $\frac{2\mu_0 Iqv}{\pi d}$ (D) 0

38. X-rays are produced in an X-ray tube operating at a given accelerating voltage. The wavelength of the continuous X-rays has values from :

(A) 0 to ∞

(B) λ_{min} to ∞ where $\lambda_{min} > 0$

(C) 0 to λ_{max} where $\lambda_{max} < \infty$

(D) λ_{min} to λ_{max} Where $0 < \lambda_{min} < \lambda_{max} < \infty$

39. A particle of mass m is executing oscillations about the origin on the x-axis. Its potential energy is $U(x) = k\,|x|^3$ where k is a positive constant. If the amplitude of oscillation is a, then its time period T is :

(A) proportional to $1/\sqrt{a}$ (B) independent of a

(C) proportional to $\sqrt{a}$ (D) proportional to $a^{3/2}$

40. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately :

(A) 540 nm (B) 400 nm

(C) 310 nm (D) 220 nm

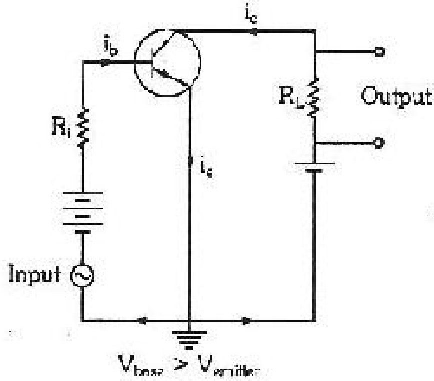
ANSWERS

1. (A), (C)	**2.** (C)	**3.** (B)	**4.** (C), (D)	**5.** (A)	**6.** (B)
7. (A)	**8.** (C), (D)	**9.** (C), (D)	**10.** (D)	**11.** (A), (D)	**12.** (D)
13. (D)	**14.** (B), (C)	**15.** (B), (C)	**16.** (C)	**17.** (B)	**18.** (D)
19. (C)	**20.** (B)	**21.** (D)	**22.** (A)	**23.** (D)	**24.** (D)
25. (B)	**26.** (A), (C)	**27.** (A)	**28.** (A), (B), (C)		**29.** (B), (C)
30. (C)	**31.** (A)	**32.** (A), (C)	**33.** (A), (C)	**34.** (D)	**35.** (B), (C)
36. (A), (B), (C), (D)		**37.** (D)	**38.** (B)	**39.** (A)	**40.** (C)

SOLUTIONS

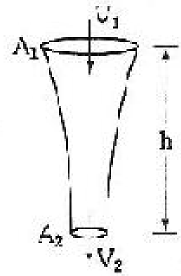
1. (A), (C)

The circuit of a common emitter amplifier is as shown below :



This has been shown a n-p-n transistor. Therefore, base-emitter are forward biased and Input signal is connected between base and emitter.

2. (C)



From conservation of energy

$$V_2^2 = V_1^2 + 2gh \qquad ...(1)$$

[can also be found by applying Bernouilli's theorem between 1 and 2]

From continuity equation

$$A_1V_1 = A_2V_2 ; V_2 = \left(\frac{A_1}{A_2}\right) V_1 \qquad ...(2)$$

Substituting value of V_2 from equation (2) in equation (1)

$$\frac{A_1^2}{A_2^2} . V_1^2 = V_1^2 + 2gh$$

or
$$A_2^2 = \frac{A_1^2V_1^2}{V_1^2 + 2gh}$$

$\therefore$
$$A_2 = \frac{A_1V_1}{\sqrt{V_1^2 + 2gh}}$$

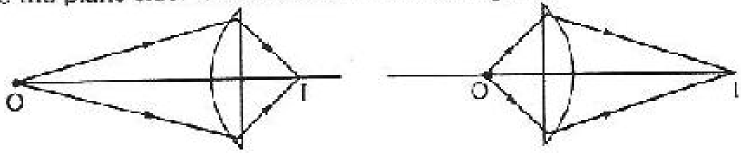
Substituting the given values

$$A_2 = \frac{(10^{-4}\ m^2)(1.0\ m/s)}{\sqrt{(1.0 m/s)^2 + 2(10)(0.15)}}$$

$$A_2 = 5.0 \times 10^{-5}\ m^2$$

3. (B)

In general spherical aberration is minimum when the total deviation produced by the system is equally divided on all refracting surfaces. A planoconvex lens is used for this purpose. In order that the total deviation be equally divided on two surfaces, it is essential that more parallel beam (of the incident and refracted) be incident on the convex side. Thus when the object is far away from the lens, incident rays will be more parallel than the refracted rays, therefore, the object should face the convex side, but if the object is near the lens, the object should face the plane side. This has been shown in figure.



4. (C, D)

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\bar{v} = \sqrt{\frac{8}{\pi}\cdot\frac{RT}{M}} \approx \sqrt{\frac{2.5RT}{M}} \text{ and}$$

$$v_p = \sqrt{\frac{2RT}{M}}$$

From these expressions we can see that

$$v_p < \bar{v} < v_{rms}$$

secondly, $v_{rms} = \sqrt{\frac{3}{2}}v_p$

and average kinetic energy of a gas molecule $= \frac{1}{2}mv_{rms}^2$

$$= \frac{1}{2}m\left(\sqrt{\frac{3}{2}}\,v_p\right)^2 = \frac{3}{4}mv_p^2$$

5. (A)

Average kinetic energy per molecule per degree of freedom $= \frac{1}{2}kT$. Since both the gases are diatomic and at same temperature (300 K), both will have the same number of rotational degree of freedom i.e. two. Therefore, both the gases will have the same average rotational kinetic energy per molecule ($= 2 \times \frac{1}{2}kT$ or kT)

Thus ratio will be 1 : 1

6. (B)

Mass per unit length of the string, $m = \frac{10^{-2}}{0.4}$

$$= 2.5 \times 10^{-2} \text{ kg/m}$$

$\therefore$ Velocity of wave in the string, $V = \sqrt{T/m} = \sqrt{\frac{1.6}{2.5 \times 10^{-2}}}$

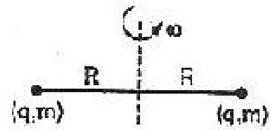
$$V = 8 \text{m/s}$$

For constructive interference between successive pulses :

$$\Delta t_{min} = \frac{2l}{V} = \frac{(2)(0.4)}{8} = 0.10 \text{ s}$$

(After two reflections, the wave pulse is in same phase as it was produced since in one reflection its phase changes by π, and if at this moment next identical pulse is produced, then constructive interference will be obtained.)

7. (A)



current, i = (frequency) (charge)

$$= \left(\frac{\omega}{2\pi}\right)(2q) = \frac{q\omega}{\pi}$$

Magnetic moment, M = (i) (A)

$$= \left(\frac{q\omega}{\pi}\right)(\pi R^2)$$

$$= (q\omega R^2)$$

Angular momentum, $L = 2I\omega = 2(mR^2)\omega$

$$\therefore \quad \frac{M}{L} = \frac{q\omega R^2}{2(mR^2)\omega} = \frac{q}{2m}$$

8. (C, D)

For total internal reflection to take place :

Angle of incidence, i > critical angle, θ_c

or $\quad \sin i > \sin\theta_c$

or $\quad \sin 45° > \frac{1}{n}$

or $\quad \frac{1}{\sqrt{2}} > \frac{1}{n}$

or $\quad n > \sqrt{2}$

or $\quad n > 1.414$

Therefore, possible values of n can be 1.5 or 1.6 in the given options.

9. (C, D)

Due to mass defect (which is finally responsible for the binding energy of the nucleus), mass of a nucleus is always less than the sum of masses of its constituent particles.

$^{20}_{10}$Ne is made up of 10 protons plus 10 neutrons. Therefore, mass of $^{20}_{10}$Ne nucleus

$$M_1 < 10(m_p + m_n)$$

Also, heavier the nucleus, more is the mass defect

Thus, $\quad 20(m_n + m_p) - M_2 > 10(m_p + m_n) - M_1$

or $\quad 10(m_p + m_n) > M_2 - M_1$

or $\quad M_2 < M_1 + 10(m_p + m_n)$

Now since $\quad M_1 < 10(m_p + m_n)$

$\therefore \quad M_2 < 2M_1$

10. (D)

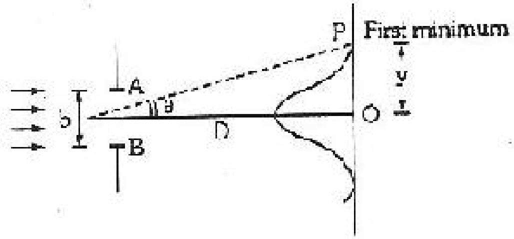
At first minima, $b \sin\theta = \lambda$

or $\qquad b\,\theta = \lambda$

or $\qquad b\left(\dfrac{y}{D}\right) = \lambda$

or $\qquad y = \dfrac{\lambda D}{b}$

or $\qquad \dfrac{yb}{D} = \lambda \qquad \ldots (1) \qquad \sin\theta \approx \theta$



Now at P (First minima) path difference between the rays reaching from two edges (A and B) will be

$$\Delta x = \frac{yb}{D} \qquad \text{(Compare with } \Delta x = \frac{yd}{D} \text{ in YDSE)}$$

or $\qquad \Delta x = \lambda \qquad$ (From 1)

Corresponding phase difference (ϕ) will be

$$\phi = \left(\frac{2\pi}{\lambda}\right) . \Delta x$$

$$\phi = \frac{2\pi}{\lambda} . \lambda$$

$$\phi = 2\pi$$

11. (A, D)

Time period, $T_n = \dfrac{2\pi r_n}{V_n} \qquad$ (in n^{th} state)

i.e. $T_n \propto \dfrac{r_n}{V_n}$

But $\qquad r_n \propto n^2$

and $\qquad V_n \propto \dfrac{1}{n}$

Therefore $\qquad T_n \propto n^3$

Given $\qquad T_{n_1} = 8\,T_{n_2}$

Hence $\qquad n_1 = 2n_2$

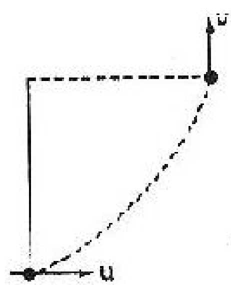
Therefore, options (A) and (D) are correct.

12. (D)

From energy conservation

$$v^2 = u^2 - 2gL \qquad \ldots (1)$$

Now since the two velocity vectors shown in figure are mutually perpendicular, hence the magnitude of change of velocity will be given by



$$|\Delta \vec{v}| = \sqrt{u^2 + v^2}$$

Substituting value of v^2 from equation (1)

$$|\Delta \vec{v}| = \sqrt{u^2 + u^2 - 2gL}$$

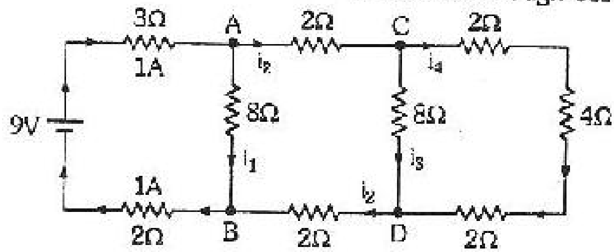
$$|\Delta \vec{v}| = \sqrt{2(u^2 - gL)}$$

13. (D)

Net resistance of the circuit is 9 Ω

∴ Current drawn from the battery, $i = \frac{9}{9} = 1\ A$

= current through 3Ω resistor



Potential difference between A and B is

$$V_A - V_B = 9 - 1(3+2) = 4V = 8i_1$$

∴ $i_1 = 0.5A$

∴ $i_2 = 1 - i_1 = 0.5\ A$

Similarly, potential difference between C and D

$$V_C - V_D = (V_A - V_B) - i_2(2+2)$$
$$= (4) - 4i_2$$
$$= 4 - 4(0.5)$$
$$= 2V = 8i_3$$

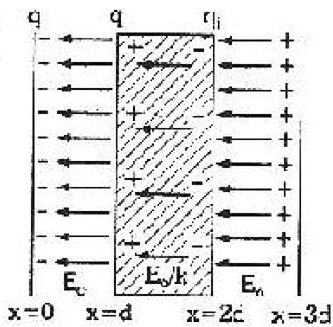
∴ $i_3 = 0.25A$

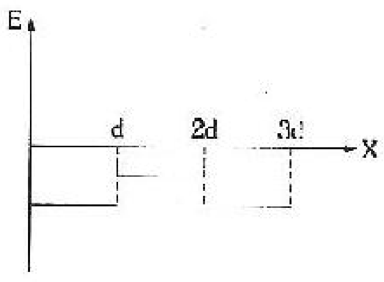
Therefore $i_4 = i_2 - i_3 = 0.5 - 0.25$

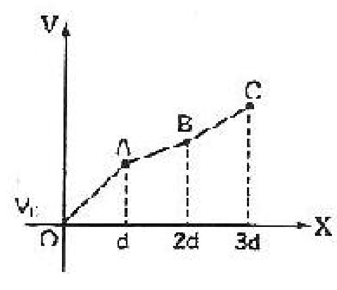
$i_4 = 0.25\ A$

14. (B,C)

The magnitude and direction of electric field at different points are shown in figure. The direction of the electric field remains the same. Hence option (B) is correct. Similarly, electric lines always flow from higher to lower potential, therefore, electric potential increases continuously as we move from x = 0 to x = 3d. Therefore, option (C) is also correct. The variation of electric field (E) and potential (V) with x will be as follows :



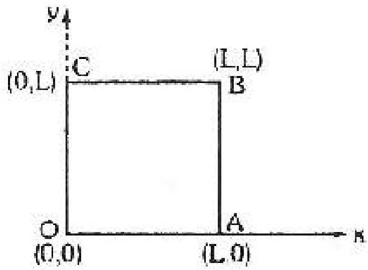




OA || BC and $(Slope)_{OA} > (Slope)_{AB}$

Because $E_{o-d} = E_{2d-3d}$ and $E_{o-d} > E_{d-2d}$

15. (B), (C)



Since the edges are clamped, displacement of the edges u (x, y) = 0 for

Line	OA	i.e.	y = 0,	$0 \le x \le L$
	AB	i.e.	x = L,	$0 \le y \le L$
	BC	i.e.	y = L,	$0 \le x \le L$
	OC	i.e.	x = 0,	$0 \le y \le L$

The above conditions are satisfied only in alternatives (B) and (C).

Note that u (x, y) = 0, for all four values e.g. in alternative (D), u (x, y) = 0 for y = 0, y = L but it is not zero for x = 0 or x = L. Similarly in option (A), u (x, y) = 0 at x = L, y = L but it is not zero for x = 0 or y = 0, while in options (B) and (C), u (x, y) = 0 for x = 0, y = 0, x = L and y = L

16. (C)

$$dW = \vec{F} \cdot \vec{ds} \text{ where } \vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

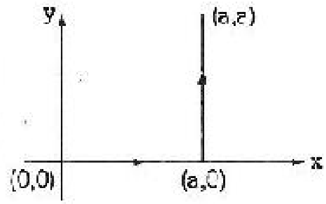
and $$\vec{F} = -K(y\hat{i} + x\hat{j})$$

$\therefore$ $$dW = -K(ydx + xdy) = -k\, d(xy)$$

$\therefore$ $$W = \int_{(0,0)}^{(a,a)} dW = -K \int_{(0,0)}^{(a,a)} d(xy) = -K[xy]_{(0,0)}^{(a,a)}$$

$$W = -Ka^2$$

Alternate Solution



While moving from (0, 0) to (a, 0) along positive x-axis, y = 0 $\therefore \vec{F} = -Kx\hat{j}$ *i.e.* force is in negative y-direction while the displacement is in positive x-direction. Therefore, $W_1 = 0$

(Force ⊥ displacement)

Then it moves from (a, 0) to (a, a) along a line parallel to y axis (x = + a). During this

$$\vec{F} = -K(y\hat{i} + a\hat{j})$$

The first component of force, $-ky\hat{i}$ will not contribute any work because this component is along negative x-direction $(-\hat{i})$ while displacement is in positive y-direction [(a, 0) to (a, a)].

The second component of force *i.e.* $-Ka\hat{j}$ will perform negative work

$$W_2 = (-ka)(a) = -Ka^2 \quad \left[\begin{matrix}\vec{F}_y = -Ka\hat{j} \\ \vec{s} = a\hat{j}\end{matrix}\right]$$

$\therefore$ $$W = w_1 + w_2 = -Ka^2$$

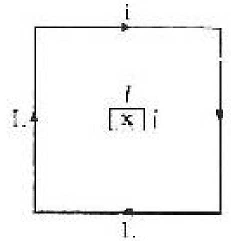
→ *The given force is conservative in nature. Therefore, work done is path independent. It depends only on initial and final positions. Therefore first method is brief and correct.*

17. (B)

Magnetic field produced by a current i in a large square loop at its centre,

$$B \propto \frac{i}{L}$$

say $$B = K\frac{i}{L}$$



∴ Magnetic flux linked with smaller loop,

$$\phi = B.S$$

$$\phi = \left(K\frac{i}{L}\right)(l^2)$$

Therefore, the mutual inductance

$$M = \frac{\phi}{i} = K\frac{l^2}{L} \qquad \text{or} \qquad M \propto \frac{l^2}{L}$$

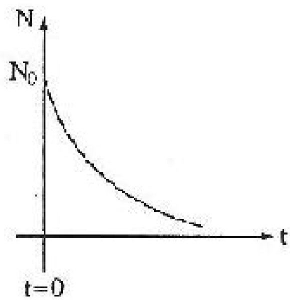
→ *Dimensions of self inductance (L) or mutual inductance (M) are :*

$$[\text{Mutual inductance}] = [\text{Self inductance}]$$
$$= [\mu_0]\,[\text{length}]$$

Similarly, Dimensions of capacitance are :

$$[\text{capacitance}] = [\epsilon_0]\,[\text{Length}]$$

From this point of view options B and D may be correct.



18. (D)

Number of nuclei decreases exponentially

$$N = N_0 e^{-\lambda t} \text{ and}$$

Rate of decay $\left(-\frac{dN}{dt}\right) = \lambda N$

Therefore, decay process lasts upto $t = \infty$. Therefore, a given nucleus may decay at any time after $t = 0$

19. (C)

Process is isothermal. Therefore, T = constant. $\left(P \propto \frac{1}{V}\right)$ volume is increasing, therefore, pressure will decrease.

In chamber A →

$$\Delta P = (P_A)_i - (P_A)_f = \frac{n_A RT}{V} - \frac{n_A RT}{2V} = \frac{n_A RT}{2V} \qquad ...(1)$$

In chamber B →

$$1.5\,\Delta P = (P_B)_i - (P_B)_f = \frac{n_B RT}{V} - \frac{n_B RT}{2V} = \frac{n_B RT}{2V} \qquad ...(2)$$

From (1) and (2)

$$\frac{n_A}{n_B} = \frac{1}{1.5} = \frac{2}{3}$$

or $$\frac{m_A/M}{m_B/M} = \frac{2}{3}$$

or $$\frac{m_A}{m_B} = \frac{2}{3}$$

or $$3m_A = 2m_B$$

20. (B)

In isothermal process

$$PV = \text{constant}$$

$\therefore$ $$PdV + VdP = 0$$

or $$\left(\frac{dP}{dV}\right) = -\left(\frac{P}{V}\right)$$

$\therefore$ Bulk modulus, $$B = -\left(\frac{dP}{dV/V}\right) = -\left(\frac{dP}{dV}\right)V$$

$\therefore$ $$B = -\left[\left(-\frac{P}{V}\right)V\right] = P$$

$\therefore$ $$B = P$$

→ *Adiabatic bulk modulus is given by $B = \gamma P$*

21. (D)

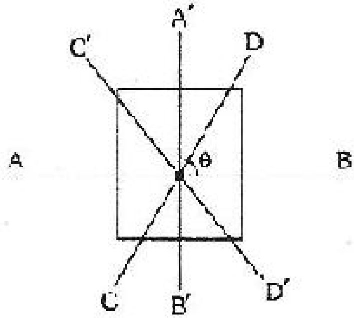
Potential at origin will be given by

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{x_0} - \frac{1}{2x_0} + \frac{1}{3x_0} - \frac{1}{4}x_0 + \ldots\right]$$

$$= \frac{q}{4\pi\epsilon_0}\cdot\frac{1}{x_0}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\right]$$

$$= \frac{q}{4\pi\epsilon_0 x_0}\ln(2)$$

22. (A)



$A'B' \perp AB$ and $C'D' \perp CD$

From symmetry $I_{AB} = I_{A'B'}$

and $I_{CD} = I_{C'D'}$

From theorem of perpendicular axes,

$$I_{ZZ} = I_{AB} + I_{A'B'} = I_{CD} + I_{C'D'}$$
$$= 2I_{AB} = 2I_{CD}$$

$\therefore$ $$I_{AB} = I_{CD}$$

Alternate Solution

The relation between I_{AB} and I_{CD} should be true for all values of θ.

and at $\theta = 0$, $I_{CD} = I_{AB}$

Similarly at $\theta = \pi/2$, $I_{CD} = I_{AB}$ (by symmetry)

Keeping these things in mind, only option (A) is correct.

23. (D)

A is free to move, therefore, heat will be supplied at constant pressure

$\therefore \quad dQ_A = nC_P dT_A \quad \ldots(1)$

B is held fixed, therefore heat will be supplied at constant volume.

$\therefore \quad dQ_B = nC_V dT_B \quad \ldots(2)$

But $\quad dQ_A = dQ_B \quad$ (Given)

$\therefore \quad nC_P dT_A = nC_V dT_B$

$\therefore \quad dT_B = \left(\frac{C_P}{C_V}\right) dT_A$

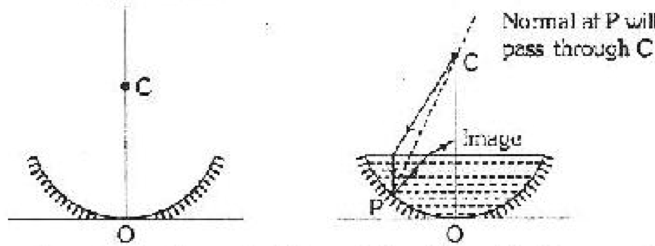
$= \gamma\,(dT_A) \quad \gamma = 1.4$ (diatomic)

$dT_A = 30K$

$= (1.4)\,(30K)$

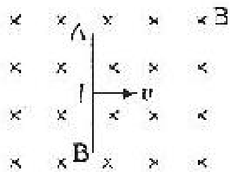
$\therefore \quad dT_B = 42K$

24. (D)



The ray diagram is shown in figure. Therefore, the image will be real and between C and D.

25. (B)



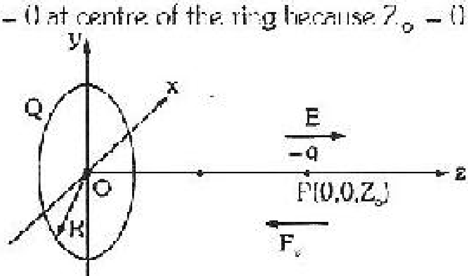
A motional emf, $e = Blv$ is induced in the rod. Or we can say a potential difference is induced between the two ends of the rod AB, with A at higher potential and B at lower potential. Due to this potential difference, there is an electric field in the rod.

26. (A, C)

Let Q be the charge on the ring, the negative charge $-q$ is released from point $P\,(0, 0, Z_0)$. The electric field at P due to the charged ring will be along positive z-axis and its magnitude will be

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{QZ_0}{(R^2 + Z_0^2)^{3/2}}$$

$E = 0$ at centre of the ring because $Z_0 = 0$



Therefore, force on charge P will be towards centre as shown, and its magnitude is

$$F_e = qE = \frac{1}{4\pi\epsilon_0} \cdot \frac{Qq}{(R^2 + Z_0^2)^{3/2}} \cdot Z_0 \qquad \ldots(1)$$

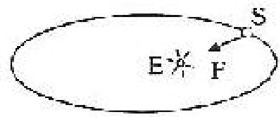
Similarly, when it crosses the origin, the force is again towards centre O. Thus the motion of the particle is periodic for all values of Z_0 lying between 0 and ∞.

Secondly if $Z_0 << R$, $(R_2 + Z_0^2)^{3/2} \approx R^3$

$$F_e \approx \frac{1}{4\pi\epsilon_0} \cdot \frac{Qq}{R^3} \cdot Z_0 \qquad \text{(From equation 1)}$$

i.e. the restoring force $F_e \propto -Z_0$. Hence the motion of the particle will be simple harmonic. (Here negative sign implies that the force is towards its mean position.)

27. (A)



Force on satellite is always towards earth, therefore, acceleration of satellite S is always directed towards centre of the earth. Net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about centre of earth is constant throughout. Since the force F is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.

28. (A, B, C)

(A) $\vec{\tau} = \vec{A} \times \vec{L}$

i.e. $\frac{d\vec{L}}{dt} = \vec{A} \times \vec{L}$

This relation implies that $\frac{d\vec{L}}{dt}$ is perpendicular to both $\vec{A}$ and $\vec{L}$. Therefore, option (A) is correct.

(C) $\vec{L} \cdot \vec{L} = L^2$ Here

Differentiating with repect to time, we get

$$\vec{L} \cdot \frac{d\vec{L}}{dt} + \frac{d\vec{L}}{dt} \cdot \vec{L} = 2L\frac{dL}{dt}$$

$$\Rightarrow \qquad 2\vec{L} \cdot \frac{d\vec{L}}{dt} = 2L\frac{dL}{dt} \qquad \ldots(1)$$

But since $\vec{L} \perp \frac{d\vec{L}}{dt}$

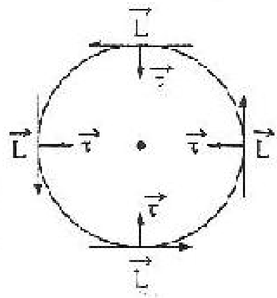
$$\therefore \qquad \vec{L} \cdot \frac{d\vec{L}}{dt} = 0$$

Therefore, from equation (1) $\frac{dL}{dt} = 0$

or magnitude of $\vec{L}$ i.e. L does not change with time.

(B) So far we are confirm about two points :

(1) $\vec{\tau}$ or $\frac{d\vec{L}}{dt} \perp \vec{L}$ and



(2) $|\vec{L}|$ or L is not changing with time, therefore it is a case when direction of $\vec{L}$ is changing but its magnitude is constant and $\vec{\tau}$ is perpendicular to $\vec{L}$ at all points.

This can be written as :

If $\vec{L} = (a \cos \theta)\,\vec{i} + (a \sin \theta)\,\vec{j}$ Here a = positive constant

then $\vec{\tau} = (a \sin \theta)\,\hat{i} - (a \cos \theta)\,\hat{j}$

so that $\vec{L} \cdot \vec{\tau} = 0$ and $\vec{L} \perp \vec{\tau}$

Now $\vec{A}$ is a constant vector and it is always perpendicular to $\vec{\tau}$. Thus $\vec{A}$ can be written as

$$\vec{A} = A\hat{k}$$

We can see that $\vec{L} \cdot \vec{A} = 0$

i.e. $\vec{L} \perp \vec{A}$ also.

Thus we can say that component of $\vec{L}$ along $\vec{A}$ is zero or component of $\vec{L}$ along $\vec{A}$ is always constant.

Finally we conclude $\vec{\tau}$, $\vec{A}$ and $\vec{L}$ are always mutually perpendicular.

29. (B, C)

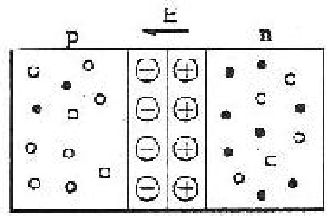
There is a decrease is volume during melting on an ice slab at 273K. Therefore, negative work is done by ice-water system on the atmosphere or positive work is done on the ice-water system by the atmosphere. Hence option (B) is correct. Secondly heat is absorbed during melting (i.e. dQ is positive) and as we have seen, work done by ice-water system is negative (dW is negative). Therefore, from first law of thermodynamics

$$dU = dQ - dW$$

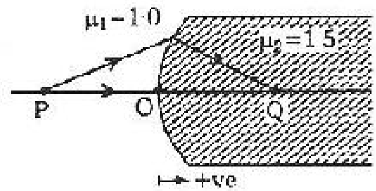
change in internal energy of ice water system, dU will be positive or internal energy will increase.

30. (C)

At junction a potential barrier/depletion layer is formed as shown, with n-side at higher potential and p-side at lower potential. Therefore, there is an electric field at the junction directed from the n-side to p-side.



31. (A)



Let us say PO = OQ = X

Applying

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Substituting the values with sign

$$\frac{1.5}{+X} - \frac{1.0}{-X} = \frac{1.5 - 1.0}{+R}$$

(Distances are measured from O and are taken as positive in the direction of ray of light)

$\therefore$ $$\frac{2.5}{X} = \frac{0.5}{R}$$

$\therefore$ $$X = 5R$$

32. (A, C)

Inside the sphere

$$E = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{R^3} r$$

$\Rightarrow$ $E \propto r$ for $r \le R$

i.e. E at centre = 0 $(r = 0)$

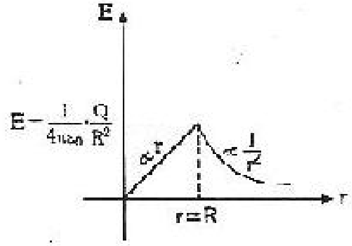
and E at surface $= \dfrac{1}{4\pi \epsilon_0} \cdot \dfrac{Q}{R^2}$ $(r = R)$

Outside the sphere

$$E = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q}{r^2} \qquad r \ge R$$

or $$E \propto \frac{1}{r^2}$$

Thus variation of electric field (E) with distance (r) from the centre will be as follows :



33. (A, C)

Maximum speed of any point on the string = $a\omega$

$$= a(2\pi f)$$

$$= v/10 \qquad \text{(Given)}$$

$$= 10/10 \qquad (v = 10 \text{ m/s})$$

$$= 1$$

$$\therefore \quad 2\pi af = 1$$

$$\therefore \quad f = \frac{1}{2\pi a}$$

$$a = 10^{-3} \text{ m} \qquad \text{(Given)}$$

$$\therefore \quad f = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi} \text{ Hz}$$

Speed of wave $v = f\lambda$

$$\therefore \quad (10 \text{ m/s}) = \left(\frac{10^3}{2\pi} S^{-1}\right)\lambda$$

$$\therefore \quad \lambda = 2\pi \times 10^{-2} \text{ m}$$

34. (D)

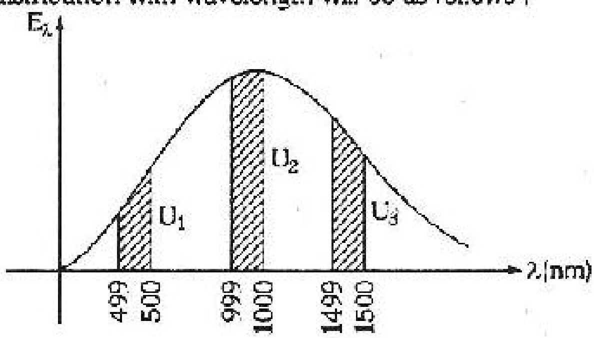
Wein's displacement law is :

$$\lambda_m T = b \qquad (b = \text{wein's constant})$$

$$\therefore \quad \lambda_m = \frac{b}{T} = \frac{2.88 \times 10^6 \text{ nm k}}{2880 \text{ nm}}$$

$$\therefore \quad \lambda_m = 1000 \text{ nm}$$

Energy distribution with wavelength will be as follows :



λ =1000nm

From the graph it is clear that

$$U_2 > U_1$$

(In fact U_2 is maximum)

35. (B,C)

$$F = \frac{1}{4\pi \epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$\therefore \quad [\epsilon_0] = \frac{[q_1][q_2]}{[F][r^2]} = \frac{[IT]^2}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4I^2]$$

Speed of light, $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

$$\therefore \quad [\mu_0] = \frac{1}{[\epsilon_0][c]^2}$$

$$= \frac{1}{[M^{-1}L^{-3}T^{4}I^{2}][LT^{-1}]^2}$$

$$= [MLT^{-2}I^{-2}]$$

36. (A, B, C, D)

(A) $L = \frac{\phi}{i}$ or Henry = $\frac{\text{Weber}}{\text{Ampere}}$

(B) $e = -L\left(\frac{di}{dt}\right)$

$\therefore$ $L = -\frac{e}{(di/dt)}$ or Henry = $\frac{\text{Volt-second}}{\text{Ampere}}$

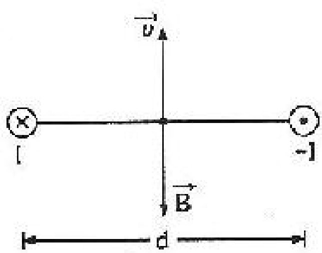
(C) $U = \frac{1}{2}Li^2$

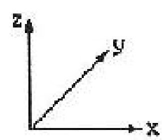
$\therefore$ $L = \frac{2U}{i^2}$ or Henry = $\frac{\text{Joule}}{(\text{Ampere})^2}$

(D) $U = \frac{1}{2}Li^2 = i^2Rt$

$\therefore$ $L = R.t$ or Henry = ohm-second

37. (D)





Wires are in x-y plane and velocity in z-direction

Net magnetic field due to both the wires will be downward as shown above.

Since angle between $\vec{v}$ and $\vec{B}$ is 180°

Therefore, magnetic force

$$\vec{F}_m = q(\vec{v} \times \vec{B}) = 0$$

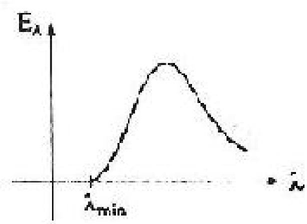
38. (B)

The continuous x-ray spectrum is shown in figure.

All wavelengths > λ_{min} are found, where

$$\lambda_{min} = \frac{12375}{V \text{ (in volts)}} \text{Å}$$

Here V is the applied voltage.



39. (A)

$$U(x) = k\,|x|^3$$

$\therefore$
$$[k] = \frac{[U]}{[x^3]} = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$$

Now time period may depend on

$T \propto (\text{mass})^x\ (\text{amplitude})^y\ (k)^z$

$\therefore$
$$[M^0L^0T] = [M]^x\ [L]^y\ [ML^{-1}T^{-2}]^z$$
$$= [M^{x+z}\ L^{y-z}\ T^{-2z}]$$

Equating the powers, we get

$$-2z = 1 \quad \text{or} \quad z = -1/2$$
$$y - z = 0 \quad \text{or} \quad y = z = -1/2$$

Hence $\quad T \propto (\text{amplitude})^{-1/2}$

$$\propto (a)^{-1/2}$$

or
$$T \propto \frac{1}{\sqrt{a}}$$

40. (C)

$$\lambda\ (\text{in } A^\circ) = \frac{12375}{W\,(eV)} = \frac{12375}{4.0}\,\text{Å} \approx 3093\ \text{Å}$$

or $\quad \lambda \approx 309.3$ nm $\quad$ or $\quad$ 310 nm.

⟶ $\lambda\ (in\ A^\circ) = \dfrac{12375}{W(eV)}$ *comes from* $W = \dfrac{hc}{\lambda}$